

1966; Okaya & Bednowitz, 1967). The principal phase relation used in this procedure is the Σ_2 formula of Hauptman & Karle (1953):

$$sE_{\mathbf{H}} \approx s \Sigma E_{\mathbf{K}} E_{\mathbf{H}-\mathbf{K}} \quad (2)$$

where s means 'sign of'. The right hand side or the 'double product sum' is modified in our procedure so that the residue of the dominant symbol determines the correct symbol assignment. The dominant symbol is the one whose double product sum is greater than the sum of all other double products associated with all other symbols. The difference between the double product sum of the dominant symbol and the sum of the double product of all other symbols is the residue (R_s). (R_s) must be less than or equal to the absolute magnitude of the double product sum when the actual signs (+ or -) are used. By modifying the usual probability formula (Karle & Karle, 1963; Woolfson, 1954) to

$$P_s(E_{\mathbf{H}}) \geq \frac{1}{2} + \frac{1}{2} \tanh \frac{\sigma_3}{\sigma_2^{3/2}} |E_{\mathbf{H}}| R_s \quad (3)$$

automated decisions could be made on a reasonably quantitative basis.

In order to initiate the automatic phase determining procedure it is necessary to choose a set of starting phases. For space group $P2_1/c$ these three reflections are given arbitrary signs to fix the origin. Additional reflections with relatively large E factors, and with many interactions with other (strong) terms, are then assigned symbolic phases. The interaction list is usually computed before assigning the first set of phases. The starting set can be augmented by using additional phase relations. For example, for this structure the Σ_1 relation (4) (Hauptman & Karle, 1953),

$$s(E_{2\mathbf{H}}) \approx s(E_{\mathbf{H}}^2 - 1) \quad (4)$$

was used to establish a positive sign for the reflection (468). The starting set used in the phase determination is listed in Table 1.

Table 1. Starting set of phases

hkl	E	Symbol	
1 5 $\bar{3}$	3.33	(+)	} arbitrarily assigned to fix the origin
2 3 4	3.11	(+)	
5 3 $\bar{6}$	2.99	(+)	
5 3 $\bar{2}$	3.09	a	} initial assignments
3 3 $\bar{1}\bar{2}$	2.68	b	
4 6 8	3.71	+	} determined from Σ_1 relation additional assignment
6 2 0	2.76	c	

Using the starting set of six reflections and applying the symbolic addition procedure manually to the 68 reflections with $|E| > 2.0$, 11 signs and 16 symbols were determined. In the next stage 108 reflections with $|E| > 1.8$ were analyzed and yielded a cumulative total of 26 signs and 44 symbols. In order to facilitate the rapid development of more symbolic assignments, the symbol (c) was then assigned to (620).

In the second pass through the computer 199 reflections with $|E| > 1.50$ were examined. A total of 42 signs and 123 symbols were known after this pass. For the third pass the minimum $|E|$ was reduced to 1.2 and 318 reflections were used and 49 signs and 193 symbols were established.

There were many indications that ($b=c$) ($a=-$) and ($b=+$) were the most probable sign assignments. Although additional phases could undoubtedly have been determined by substituting these signs for the symbols in the phase determining procedure and continuing that process, we decided to terminate the phase analysis and prepare E maps based on signs and symbols determined at the end of the third computer pass. The nine non-hydrogen atoms in the molecule were clearly revealed

Table 2. Atomic coordinates and thermal parameters

Estimated standard errors of the coordinates, the b_{ij} and their standard errors have been multiplied by 10^4 . The thermal parameters are given by: $h^2b_{11} + k^2b_{22} + l^2b_{33} + 2hkb_{12} + 2hll_{13} + 2klb_{23}$.

	x	σ_x	y	σ_y	z	σ_z	B	b_{11}	b_{22}	b_{33}	b_{12}	b_{13}	b_{23}
O(1)	0.6506	2	0.2975	2	0.0921	1	178 (3)	126 (4)	73 (1)	43 (3)	69 (1)	17 (2)	
O(2)	0.5396	2	0.0183	2	0.1531	1	123 (2)	165 (4)	41 (1)	-2 (2)	31 (1)	-10 (1)	
C(1)	0.6327	2	0.0932	3	0.1065	1	79 (3)	150 (6)	33 (1)	25 (3)	10 (1)	-2 (2)	
C(2)	0.7333	2	0.9210	4	0.0688	1	93 (3)	174 (6)	40 (1)	29 (3)	17 (1)	-12 (2)	
C(3)	0.9012	2	0.9039	4	0.1285	2	102 (3)	263 (8)	42 (1)	61 (4)	10 (1)	-21 (2)	
C(4)	0.0047	2	0.7282	4	0.0951	1	95 (2)	195 (7)	46 (1)	37 (3)	17 (1)	-14 (2)	
C(5)	0.1775	2	0.7302	4	0.1480	1	103 (3)	182 (6)	39 (1)	46 (3)	18 (1)	-17 (2)	
C(6)	0.2791	2	0.5532	4	0.1152	1	92 (3)	157 (6)	43 (1)	24 (3)	18 (1)	-10 (2)	
N	0.4502	2	0.5745	3	0.1627	1	99 (2)	143 (5)	42 (1)	28 (3)	24 (1)	7 (2)	
H(1)	0.7438	26	0.9662	40	0.0093	16	3.3 (5)	\AA^2					
H(2)	0.6808	27	0.7729	45	0.0650	16	3.4 (4)						
H(3)	0.9639	29	0.0555	48	0.1367	16	4.1 (5)						
H(4)	0.8944	30	0.8790	50	0.1900	20	4.9 (6)						
H(5)	0.0081	30	0.7667	50	0.0275	18	4.6 (6)						
H(6)	0.9617	24	0.5975	42	0.1022	14	2.4 (4)						
H(7)	0.1836	30	0.7120	46	0.2157	19	4.7 (6)						
H(8)	0.2214	30	0.8884	49	0.1404	17	4.4 (6)						
H(9)	0.2414	26	0.4149	42	0.1207	14	2.8 (5)						
H(10)	0.2829	27	0.5877	44	0.0518	17	3.5 (5)						
H(11)	0.4551	25	0.5571	40	0.2241	16	3.2 (5)						
H(12)	0.4939	26	0.7142	48	0.1562	15	3.4 (5)						
H(13)	0.5181	27	0.4651	42	0.1361	15	3.3 (5)						

Table 3. $|F_{\text{Obs}}|$ and $|F_{\text{Calc}}|$

L	F _{Obs}	F _{Calc}	L	F _{Obs}	F _{Calc}	L	F _{Obs}	F _{Calc}	L	F _{Obs}	F _{Calc}	L	F _{Obs}	F _{Calc}	L	F _{Obs}	F _{Calc}
1	H = 0	1	6 38	39	13 25	27	-10 0*	2	5 44	45	10 32	32	-7 240	231	0 63	60	
2	(K = 0)	7	0*	7	14 0*	2	-9 62	63	6 183	182	11 0*	2	-6 234	233	1 137	140	
3	2 135	134	8 0*	8	15 44	43	-8 63	62	7 125	125	12 23	21	-5 101	101	2 26	26	
4	6 39	38	9 69	65	16 84	80	-7 48	46	8 0*	0	11 11	11	-4 182	152	3 26	29	
5	8 147	175	10 0*	12	17 22	21	-6 28	26	9 29	28	(K = 6)	3	-3 182	173	4 34	38	
6	10 41	45					-5 102	98	10 125	129	-10 0*	34	-2 171	165	5 106	109	
7	12 72	77	(K = 7)		(K = 3)		-4 87	83	11 117	119	-9 0*	22	-1 30	14	6 0*	2	
8	14 17	18	1 64	64	-17 21	23	-3 90	92	12 0*	19	-8 0*	28	0 208	207	7 58	57	
9	16 51	54	2 60	54	-16 16	17	-2 13	13	13 21	20	-7 0*	22	1 93	92	8 81	78	
10	18 23	29	3 95	88	-15 0*	2	-1 76	71	14 47	45	-6 0*	11	2 52	52	9 27	24	
			4 0*	13	-14 19	15	0 0*	21	15 42	38	-5 65	63	3 74	71	10 0*	20	
	(K = 1)				-13 58	59	1 93	96	16 25	22	-4 52	48	4 40	43	11 0*	76	
	4 165	156	(H = 1)		-12 32	33	2 12	2			-3 0*	16	5 88	87	12 0*	50	
	5 13	52	(K = 0)		-11 0*	7	3 0*	16	(K = 3)		-2 0*	28	6 72	74			
	6 38	27	-18 64	61	-10 35	38	4 0*	33	-17 0*	12	-1 0*	83	7 0*	7	(K = 6)	3	
	7 52	56	-16 43	39	-9 77	80	5 0*	35	-16 13	17	0 0*	24	8 33	36	-10 36	33	
	8 157	156	-14 0*	0	-8 64	61	6 0*	20	-15 15	15	1 23	26	9 24	24	-9 0*	4	
	9 31	43	-12 140	136	-7 0*	1	7 0*	51	-14 0*	4	2 0*	0	10 42	44	-8 52	47	
	10 0*	2	-10 206	208	-6 95	92	8 0*	18	-13 0*	13	3 29	30	11 55	56	-7 24	28	
	11 46	53	-8 281	275	-5 0*	0	9 0*	6	-12 65	60	4 29	27	12 22	26	-6 0*	4	
	12 74	79	-6 268	260	-4 16	10			-11 45	44	5 50	55	13 13	14	-5 24	23	
	13 32	38	-4 229	225	-3 33	29			-10 25	27	6 0*	0	14 0*	9	-4 42	38	
	14 0*	7	-2 60	42	-2 52	49	(H = 2)		-9 0*	0	7 28	32	15 30	30	-3 59	60	
	15 43	49	0 401	407	-1 165	161	-18 57	53	-8 248	239	8 34	36			-2 42	40	
	16 18	23	2 367	355	0 97	103	-16 77	75	-7 54	57	9 38	42	(K = 3)		-1 32	33	
	17 0*	10	4 550	572	1 0	2	-14 60	61	-6 121	125			-17 20	20	0 33	30	
	18 0*	0	6 133	127	2 148	147	-12 99	99	-5 124	126			-16 79	71	1 75	72	
			8 86	101	3 202	205	-10 160	159	-4 423	435	-5 0*	9	-15 48	47	2 24	19	
	(K = 2)				10 0*	1	-8 122	112	-3 0*	10	0 17	13	-14 41	37	3 13	10	
	0 187	198	12 67	72	5 0*	9	-6 259	251	-2 188	192	1 0*	22	-13 45	44	4 0*	2	
	1 157	150	14 0*	8	6 0*	20	-4 105	101	-1 234	237	2 36	32	-12 160	151	5 52	43	
	2 148	130	16 48	50	7 94	99	-2 45	36	0 263	275	3 35	32	-11 120	127	6 95	90	
	3 69	52	18 0*	22	8 0*	51	0 115	113	1 103	93			-10 64	70	7 30	27	
	4 145	127			9 0*	26	2 560	552	2 27	20	(H = 3)		-9 59	66	8 29	29	
	5 227	196	(K = 1)		10 138	141	4 312	308	3 204	208	(K = 0)		-8 134	128			
	6 261	228	-18 27	27	11 21	23	6 307	301	4 0*	11	-18 32	32	-7 157	158	(K = 7)		
	7 23	19	-17 30	29	12 17	20	8 101	97	5 45	39	-16 33	31	-6 73	77	0 0*	5	
	8 63	63	-16 60	61	13 16	16	10 0*	8	6 82	87	-14 77	73	-4 0*	2			
	9 54	43	-15 40	43	14 138	134	12 57	58	7 75	69	-12 54	54	-3 80	80	(H = 4)		
	11 22	16	-14 37	41	15 0*	9	14 63	62	8 11	10	-10 166	163	-2 48	46	(K = 0)		
	12 30	31	-13 0*	1	16 0*	10	16 64	61	9 16	20	-8 21	25	-1 27	25	-18 0*	10	
	14 0*	7	-12 25	21					10 35	35	-6 279	263	0 84	87	-16 17	18	
	15 0*	7	-11 12	18	(K = 4)		(K = 1)		11 17	19	-4 17	24	1 65	64	-14 12	10	
	16 29	30	-10 0*	0	-15 45	44	-18 0*	3	12 0*	5	-2 35	43	2 0*	3	-12 72	75	
	17 64	73	-9 29	33	-14 52	50	-17 66	64	13 0*	3	0 118	115	3 0*	57	-10 55	57	
			-8 243	244	-12 0*	26	-16 29	29	14 26	26	2 721	725	4 0*	1	-8 14	36	
	(K = 3)		-7 74	79	-11 41	43	-14 0*	7	15 31	32	4 66	64	5 0*	98	-6 116	122	
	1 192	174	-6 153	153	-10 101	98	-13 0*	8	(K = 4)		6 660	692	6 38	40	-4 12	11	
	2 136	120	-5 170	169	-9 15	19	-12 107	105	-15 49	51	10 343	352	8 26	31	0 185	182	
	3 116	109	-4 494	523	-8 20	18	-11 121	114	-14 0*	23	12 27	27	9 46	47	2 265	243	
	4 59	49	-3 439	480	-7 46	43	-10 103	98	-13 0*	6	14 110	106	10 39	39	4 23	24	
	5 58	51	-2 471	502	-6 131	132	-9 92	87	-12 0*	22	16 0*	0	11 0*	6	6 11	12	
	6 48	46	-1 431	451	-5 54	94	-8 60	59	-11 0*	156			12 0*	4	8 51	41	
	7 24	22	0 398	427	-4 14	26	-7 242	224	-10 0*	7			13 0*	5	10 15	15	
	8 0*	3	1 432	464	-3 166	168	-6 231	219	-9 62	57	-18 0*	6	14 0*	2	12 57	65	
	9 49	50	2 389	391	-2 109	120	-5 207	198	-8 53	52	-17 57	57			14 32	33	
	10 44	45	3 319	319	-1 219	223	-4 176	173	-7 178	184	-16 0*	13	(K = 4)				
	11 21	23	5 60	51	-1 198	203	-3 129	109	-6 71	72	-15 44	45	-15 0*	5	(K = 1)		
	12 24	26	6 165	168	0 33	28	-2 201	191	-5 95	98	-14 22	27	-14 59	58	-18 28	27	
	14 0*	21	8 61	65	2 35	40	-1 227	208	-4 92	101	-13 31	30	-13 0*	0	-17 87	86	
	15 30	21	9 37	36	3 124	121	0 316	317	-3 54	58	-12 41	44	-12 47	45	-16 32	30	
	16 0*	3	10 56	56	4 0*	25	1 127	140	-2 137	146	-11 79	78	-11 80	82	-15 114	110	
	(K = 4)		11 0*	3	5 132	134	2 0*	10	-1 28	29	-10 14	11	-10 121	124	-14 127	125	
	0 127	130	12 0*	61	6 0*	17	3 256	248	0 83	82	-9 143	139	-9 17	15	-13 166	171	
	1 142	151	13 38	37	7 0*	36	4 736	228	1 111	115	-8 52	49	-8 74	75	-12 90	89	
	2 0*	12	14 0*	1	8 0*	10	5 0*	3	2 88	88	-7 158	150	-7 173	177	-11 139	136	
	3 92	95	15 0*	8	9 0*	36	6 0*	86	3 28	32	-6 197	186	-6 103	99	-10 118	217	
	4 67	69	16 31	30	10 0*	30	7 121	117	4 0*	13	-5 420	409	-5 92	91	-9 115	124	
	5 99	101	17 0*	17	11 0*	12	8 109	111	5 0*	130	-4 145	136	-4 65	70	-8 197	187	
	6 46	49			12 0*	21	9 73	73	6 0*	1	-3 148	138	-3 137	146	-7 98	92	
	7 47	46	(K = 2)		13 0*	37	10 97	98	7 84	82	-2 314	305	-2 0*	0	-6 154	147	
	8 0*	4	-18 0*	3	14 0*	18	11 23	23	8 76	82	-1 280	272	-1 123	127	-5 59	50	
	9 0*	5	-17 0*	6			12 67	66	9 84	84	0 107	103	0 35	37	-4 193	183	
	10 41	45	-16 34	32	(K = 5)		13 79	79	10 0*	13	1 0*	165	1 45	42	-3 38	42	
	11 25	24	-15 20	20	-13 18	17	14 73	70	11 73	75	2 0*	146	2 77	83	-2 160	156	
	12 42	44	-14 19	19	-12 17	9	15 0*	3	12 67	74	3 15	15	3 56	50	-1 74	74	
	13 62	65	-13 27	28	-11 60	57	16 22	22	13 47	43	4 96	94	4 12	10	0 136	125	
	14 30	31	-12 18	18	-10 15	14	17 30	33	14 0*	7	5 219	214	5 92	90	1 162	165	
	15 0*	7	-11 98	91	-9 0*	3			6 33	34	6 52	53	7 25	24	2 290	300	
	(K = 5)		-9 56	56	-8 85	80	(K = 2)		(K = 5)		7 0*	12	8 15	16	3 157	163	
	1 125	137	-8 151	154	-7 177	178	-18 0*	5	-13 0*	6	8 104	109	8 35	33	4 214	210	
	2 119	129	-7 0*	1	-6 74	73	-17 12	11	-12 39	36	9 72	74	9 35	33	5 202	205	
	3 0*	7	-6 214	211	-5 11	10	-15 25	22	-10 0*	12	10 0*	1	10 12	9	6 139	134	
	4 27	26	-5 319	308	-4 121	125	-14 38	37	-9 14	13	11 0*	4	11 0*	34	7 150	151	
	5 173	182	-4 346	338	-3 237	255	-13 67	68	-8 0*	12	12 72	72	12 23	24	8 107	109	
	6 31	34	-3 55	52	-2 133	34	-12 43	43	-7 79	83	13 17	14	13 12	9	9 128	131	
	7 74	79	-2 119	113	0 91	87	-11 59	54	-6 36	34	14 0*	11	14 0*	5	10 43	38	
	8 0*	11	-1 462	50													

Table 3 (cont.)

L	F#BS	FCAL	L	F#BS	FCAL	L	F#BS	FCAL	L	F#BS	FCAL	L	F#BS	FCAL	L	F#BS	FCAL	L	F#BS	FCAL							
-9	29	30	1	0*	3	2	34	35	0	22	20	-1	108	112	4	0*	1	-6	294	299	-4	53	55				
-8	27	29	2	106	102	3	41	40	1	33	31	0	76	75	5	16	12	-4	19	18	-2	93	99				
-7	20	25	3	151	148	4	162	168	2	26	47	1	21	12	6	0*	0	-2	208	224	0	24	24				
-6	70	66	4	93	93	5	31	29	3	13	12	2	113	110	7	21	24	0	40	41	2	35	35				
-5	24	26	5	13	12	6	26	28	4	24	20	3	56	53	8	38	37	2	43	45	4	0*	6				
-4	121	118	6	74	69	7	0*	46	4	14	13	9	24	13	9	24	24	4	19	19	6	0*	11				
-2	172	172	7	84	79	8	40	43	(H = 6)	5	0*	10	10	46	41	6	14	13	6	14	13	6	0*	11			
-1	0*	1	8	38	34	9	0*	5	(K = 0)	6	11	7	6	11	7	8	13	14	8	13	14	(K = 1)					
0	181	170	9	0*	11	10	17	22	-16	0*	4	7	0*	4	(K = 2)							-13	39	39			
1	129	133	11	21	22	12	21	22	-14	33	32	8	0*	1	-15	64	61	(K = 1)				-12	11	9			
2	144	144	(K = 6)	17	18	19	-12	72	71	9	0*	10	10	28	24	-14	18	14	-15	19	19	-11	0*	9			
3	34	32	-9	73	69	13	0*	14	-10	53	52	10	28	24	-13	11	8	-14	20	17	-10	43	39				
4	29	32	-8	126	123	(K = 3)	-8	141	142	-6	16	19	(K = 4)						-12	68	68	-13	0*	9			
5	161	160	-7	54	50	-16	52	54	-4	118	124	-13	35	28	-10	31	41	-11	22	20	-8	20	19				
6	111	105	-6	0*	6	-15	0*	6	-2	42	38	-12	14	11	-9	28	30	-10	21	21	-6	31	31				
7	24	25	-4	84	74	-14	0*	0	0	55	53	-11	34	31	-8	86	93	-9	50	49	-5	0*	4				
8	50	47	-3	74	73	-13	0*	11	2	45	40	-10	28	26	-7	16	17	-8	81	81	-4	30	30				
9	81	75	-2	0*	56	-12	61	60	4	0*	5	-9	0*	6	-6	63	67	-7	33	31	-3	51	53				
10	76	70	-1	0*	17	-11	16	42	6	47	48	-8	52	56	-5	80	80	-6	16	19	-2	0*	9				
11	0*	6	-1	0*	9	-10	20	22	8	56	54	-7	40	40	-4	39	40	-5	113	111	-1	43	38				
12	29	31	1	71	66	-9	57	58	10	36	35	-6	39	40	-3	35	37	-4	40	43	0	31	33				
13	24	22	2	95	89	-8	28	29	12	90	90	-5	27	24	-2	55	55	-3	46	48	1	31	32				
14	20	18	3	58	58	-7	67	64	(K = 1)	-3	0*	9	-4	111	111	-1	49	52	-2	0*	6	2	0*	12			
(K = 3)			4	28	29	-6	17	20	-5	76	80	-17	0*	8	-2	62	63	1	20	19	0	23	12	4	16	18	
-17	15	16	5	38	34	-4	72	74	-16	15	12	-2	62	63	1	20	19	0	23	12	4	16	18				
-16	42	44	(H = 5)	-3	0*	14	-15	0*	0	81	80	3	15	14	3	15	14	2	30	29	6	20	23				
-15	22	23	(K = 0)	-2	170	121	-14	0*	8	1	94	94	4	51	51	3	0*	2									
-14	13	14	-18	19	17	-1	59	60	-13	34	33	2	48	43	5	16	17	4	31	23	(K = 2)						
-13	0*	10	-16	137	149	0	82	85	-12	28	27	3	86	83	6	16	14	5	43	36	-12	25	24				
-12	50	49	-14	0*	0	1	109	108	-11	93	96	4	0*	7	7	0*	10	6	28	28	-11	77	76				
-11	77	81	-12	210	226	-10	278	260	-10	39	34	5	94	91	8	27	24	7	23	22	-10	84	82				
-10	31	30	-10	64	67	3	0*	4	-9	71	67	6	20	19	9	0*	12	8	0*	1	-9	25	22				
-9	13	7	-8	208	209	4	94	96	-8	44	45	7	11	8	10	12	12				-8	19	16				
-8	36	34	-6	118	114	5	105	112	-7	102	105	8	17	18	(K = 2)						-7	87	88				
-7	128	133	-4	130	130	6	182	186	-6	99	95	9	29	31	(K = 3)						-6	104	104				
-6	56	53	-2	128	124	7	19	16	-5	102	97	(K = 5)									-5	0*	7				
-5	23	19	0	56	55	8	48	45	-4	13	10	-11	22	23	-13	0*	5	-12	51	48	-4	0*	3				
-4	32	34	0	89	86	9	64	65	-3	70	64	-10	22	22	-12	19	20	-11	72	70	-3	43	45				
-3	76	81	4	18	18	10	72	74	-2	58	58	-10	22	22	-11	13	11	-10	31	33	-2	55	58				
-2	0*	5	6	28	26	11	20	18	-1	121	118	-9	31	34	-10	55	58	-9	21	18	-1	12	8				
-1	27	29	8	62	70	12	0*	5	0	93	84	-8	15	17	-9	31	31	-8	57	57	0	18	15				
0	109	100	8	62	70	12	0*	5	1	211	204	-7	29	30	-8	57	59	-7	35	35	1	28	25				
1	18	15	10	0*	12	(K = 4)			2	40	38	-6	37	38	-7	0*	7	-6	41	42	2	36	31				
2	35	27	12	40	43	-14	0*	0	3	74	72	-5	24	27	-6	41	43	-5	35	35	3	0*	5				
3	0*	6	14	18	19	-13	37	32	4	75	69	-4	37	37	-5	68	69	-4	27	25	4	20	18				
4	39	43	(K = 1)	-12	43	39	5	113	111	-3	47	48	-3	47	48	-4	0*	5	-3	13	14	5	46	47			
5	63	65	-18	0*	51	-11	56	57	6	52	55	-2	30	29	-3	11	10	-2	20	17	(K = 3)						
6	0*	3	-17	0*	30	-10	0*	0	7	22	20	-1	16	19	-2	48	48	-1	30	33	-10	33	31				
7	21	23	-16	0*	0	-10	19	16	8	29	26	0	34	32	-1	54	54	0	14	15	-9	26	22				
8	31	33	-15	87	82	-9	92	87	9	26	25	1	35	34	0	42	43	1	0*	6	-8	16	14				
9	43	39	-14	65	62	-8	92	87	10	0*	9	2	21	20	1	17	18	2	12	7	-7	16	14				
10	0*	5	-13	50	49	-7	48	48	11	58	56	3	0*	0	2	38	38	3	31	25	-7	20	17				
11	22	25	-12	20	24	-6	35	32	12	12	7	4	25	21	3	18	13	4	24	25	-6	77	71				
12	27	24	-11	165	160	-5	109	120	(K = 2)			5	23	23	4	0*	7	5	0*	7	-5	31	24				
13	13	10	-9	20	19	-4	42	38	-16	58	61	6	29	37	5	20	18	6	35	29	-4	25	25				
(K = 4)			-8	0*	8	-2	56	59	-15	18	16	(K = 6)									7	58	56	-3	24	23	
-15	51	52	-7	115	108	0	79	82	-14	16	14	-4	44	40	8	16	16	(K = 3)			-2	40	35				
-14	15	12	-6	153	156	1	125	124	-13	61	63	-3	0*	0	(K = 4)						-12	13	12	0	43	34	
-13	15	17	-5	106	100	2	76	78	-12	83	83	-2	58	58	-11	21	17	-9	0*	0	-11	24	20	1	36	33	
-12	26	22	-4	110	104	3	66	67	-11	0*	5	-1	26	27	-12	0*	0	-10	51	51	2	11	14				
-11	61	62	-3	82	80	4	93	107	-10	63	68	(H = 7)									-11	21	17	-9	0*	0	
-10	0*	10	-3	82	80	4	93	107	-10	63	68	(K = 0)										-10	27	26	-8	25	23
-9	79	82	-2	100	95	5	52	52	-9	40	44	(K = 1)										-9	66	71	-7	18	19
-8	84	91	-1	237	233	6	49	50	-8	0*	7	(H = 7)										-8	11	9	-6	54	55
-7	23	25	0	122	118	7	0*	0	-7	17	18	-16	11	11	-8	11	9	-6	54	55	-10	0*	6				
-6	0*	17	1	169	169	8	50	50	-6	80	85	-14	0*	5	-7	34	34	-5	26	25	-8	14	15				
-5	78	81	2	39	44	9	0*	0	-5	82	87	-12	0*	7	-6	16	15	-4	16	15	-6	0*	8				
-4	101	106	3	102	96	(K = 5)			-4	153	151	-10	61	61	-5	100	103	-3	37	37	-4	42	45				
-3	100	100	4	46	46	-12	0*	38	-2	55	54	-8	22	21	-4	16	15	-2	31	30	-2	22	25				
-2	63	62	5	102	101	-11	81	69	-1	170	171	-6	28	37	-3	52	53	-1	24	22	0	27	27				
-1	23	21	6	66	66	-10	23	14	0	234	219	-4	0*	8	-2	44	45	0	24	21	2	0*	5				
0	95	94	7	46	49	-9	0*	10	1	83	80	(K = 3)									-1	42	43	1	25	23	
1	0*</																										

on one of the maps and the signs used to compute that map were used to refine the atomic coordinates. The indicated positions of these atoms were taken as the starting points of a least-squares analysis, using the set of 242 reflections for which phases had been established. Initially, the atoms were assumed to have isotropic thermal motion. The residual (R),

$$R = \frac{\sum |F_{\text{obs}}| - |KF_{\text{calc}}|}{\sum |F_{\text{obs}}|}$$

for this limited set of data was reduced to 0.128 before including additional reflection data in the least-squares analysis. Because of limitations in the least-squares program and in the programmed data storage area, the data were split arbitrarily into two groups, each containing about 750 reflections and refined separately to residuals of 0.22 and 0.20, with only slight differences (0.001–0.0012) among the atomic coordinates derived from the two sets separately.

Refinement was then continued with the use of all the data. The isotropic refinement converged to $R = 0.15$. After the introduction of anisotropic temperature factors, R decreased to 0.12. At this point an electron density difference map was computed. A composite drawing, based on this map, revealed the locations of all thirteen hydrogen atoms in the asymmetric unit of structure. It was also clear at this stage that one hydrogen atom had been shifted from the carboxyl to the amine group to form a zwitterion. After introduction of the hydrogen atoms into the least-squares analysis with fixed and finally with variable isotropic temperature factors, the residual was reduced to 0.05 (not including the unobserved reflections). The final atomic parameters and temperature factors are listed in Table 2. The observed and calculated structure factors are listed in Table 3; the unobserved reflections (given zero weight in the least-square analysis) are indicated by asterisks. The estimation of σ used in the standard weighting scheme was:

$$\begin{aligned} F_{\text{obs}} > 10.0 & \quad \sigma = 0.1F_{\text{obs}} \\ F_{\text{obs}} \leq 10.0 & \quad \sigma = 1.0 \\ F_{\text{obs}} = 0.0 & \quad \text{weight} = 0. \end{aligned}$$

Discussion

Accuracy of the structure

The refinement of the structure was continued until changes of coordinates of non-hydrogen atoms between successive least-squares cycles were less than two per cent of their standard errors; the corresponding changes for hydrogen atoms were less than four per cent.

Standard errors of parameters were computed by inversion of the least-squares matrices. The indicated values were then multiplied by two and rounded off to the nearest significant digit before being included in this manuscript. Hamilton (1965) has emphasized the usefulness of a procedure of this sort to furnish

a measure of the accuracy, rather than the precision, of a crystal structure determination. The final value of R for the 1194 observed reflections was 5.0%.

Temperature factors

Temperature factors are listed in Tables 2 and 6. The magnitudes of atomic thermal motions along the principal axes of the thermal ellipsoids are given in Table 6 together with the r.m.s. thermal displacements of the hydrogen atoms. As expected, the thermal motions of the carbon atoms are smallest along the molecular chain direction. The magnitudes of the thermal motions of the atoms in the molecular chain are largest near the center of the chain, *i.e.* at C(3) and C(4). The hydrogen bonds at both ends of the molecules restrict the freedom of movement of atoms nearest those bonds and have least effect on atoms near the chain centers. Only axis 3 of O(1) does not fit this description; its anomalously large value may reflect the fact that it is associated with only one hydrogen bond; O(2) is involved in two such bonds.

Molecular configuration

Fig. 1 is a schematic drawing of the molecule, listing bond lengths and approximate magnitudes and orientations of the principal axes of the atomic thermal ellipsoids relative to the long molecular direction. Intramolecular bond lengths and intermolecular N–H–O distances are listed in Tables 4 and 5. As might have been expected, the two carbon–oxygen bonds in the molecule are almost equal in length, 1.235 and 1.261 Å both ± 0.005 Å; after correction for ‘riding’ of the oxygen atoms on the carbon atoms to which they are bonded (Busing & Levy, 1964) these become 1.263 and 1.271 Å. Similar results have been reported by Marsh and co-workers in their very precise analyses of the zwitterions L-lysine monochloride dihydrate (Wright & Marsh, 1962) with C–O distances of 1.246 and 1.250 Å and L-alanine (Simpson & Marsh, 1966) with 1.253 and 1.265 Å.

Table 4. Bond lengths and angles

Bond lengths*		Bond angles‡	
C(1)–O(1)	1.235 Å†	O(1)–C(1)–O(2)	124.0°
C(1)–O(2)	1.261†	O(1)–C(1)–C(2)	118.3
C(1)–C(2)	1.523	O(2)–C(1)–C(2)	117.7
C(2)–C(3)	1.520	C(1)–C(2)–C(3)	110.8
C(3)–C(4)	1.526	C(2)–C(3)–C(4)	112.7
C(4)–C(5)	1.516	C(3)–C(4)–C(5)	112.8
C(5)–C(6)	1.511	C(4)–C(5)–C(5)	112.5
(Average C–C)	1.519	C(5)–C(6)–N	110.9
		(Average C–C–C angle)	112.2°
C(6)–N	1.485		
N–H–O(1)	2.753		
N–H–O(2)	2.728		
N–H–O(2')	2.814		

* The average standard deviation of the bond lengths is 0.005 Å; the largest is 0.006 Å, the smallest 0.004 Å.

† After correction for ‘riding’ of the oxygen on C(1) these become C(1)–O(1) = 1.263, C(1)–O(2) = 1.271 Å.

‡ The standard deviations of these bond angles range from 0.3 to 0.4°.

The average length of the five carbon-carbon bonds in ϵ -aminocaproic acid is 1.519 Å. For the two crystal structures mentioned above, Marsh and co-workers reported average carbon-carbon bond lengths of 1.523 and 1.525 Å. It appears that the 'normal' single carbon-

carbon bond length, at least for molecules of this type, is in the range 1.519 to 1.525 Å.

Hahn (1957) reported an apparent lengthening of the carbon-nitrogen (C-NH₃⁺) bond in a number of zwitterions; he listed an average value of 1.503 Å com-

Table 5. Bond lengths and angles involving hydrogen atoms

Lengths*		Angles†		Angles†	
C(2)-H(1)	0.97 Å	C(1)-C(2)-H(1)	109°	N—C(6)-H(9)	112°
C(2)-H(2)	0.97	C(3)-C(2)-H(1)	108	N—C—H(10)	102
C(3)-H(3)	1.03	C(1)-C(2)-H(2)	111	H(12)-N—H(11)	106
C(3)-H(4)	0.97	C(3)-C(2)-H(2)	111	H(13)-N—H(12)	104
C(4)-H(5)	1.07	C(2)-C(3)-H(3)	110	H(11)-N—H(13)	116
C(4)-H(6)	0.92	C(4)-C(3)-H(3)	107	H(1)-C(2)-H(2)	109
C(5)-H(7)	1.03	C(2)-C(3)-H(4)	114	H(3)-C(3)-H(4)	99
C(5)-H(8)	1.02	C(4)-C(5)-H(4)	113	H(5)-C(4)-H(6)	114
C(6)-H(9)	0.89	C(3)-C(4)-H(5)	109	H(7)-C(5)-H(8)	106
C(6)-H(10)	1.00	C(5)-C(4)-H(5)	106	H(9)-C(6)-H(10)	112
(Average C-H=0.99 Å)		C(3)-C(4)-H(6)	107		
		C(5)-C(4)-H(6)	108		
N-H(11)	0.94	C(4)-C(5)-H(7)	111	(Average H-C-H	108°)
N-H(12)	0.92	C(6)-C(5)-H(7)	111	(hydrogen bonds)	
N-H(13)	1.01	C(4)-C(5)-H(8)	107	N-H(13)-O(1)	174
(Average N-H	0.96 Å)	C(6)-C(5)-H(8)	110	N-H(12)-O(2)	166
		C(5)-C(6)-H(9)	110	N-H(11)-O(4)	179
		C(5)-C(6)-H(10)	109		
		(Average C-C-H	109.5°)		

* The standard deviations of the bond lengths are 0.05 Å, except 0.06 Å for C(3)-H(3) and C(3)-H(4).

† The standard deviation of the bond angles involving one hydrogen atom is 3°; for those involving two hydrogen atoms it is 4°.

Table 6. Thermal motions and orientations of thermal ellipsoids

	R.M.S. components of thermal motion along principal axes (Å)				Angles between principal axes and vector defined by C(2)-C(6) (°)		
	Axis 1	Axis 2	Axis 3	$\sqrt{\mu^2}$ (Å)	Axis 1	Axis 2	Axis 3
O(1)	0.138	0.194	0.312	0.39	132	56	61
O(2)	0.164	0.184	0.229	0.34	127	64	49
C(1)	0.143	0.181	0.196	0.30	24	114	94
C(2)	0.148	0.196	0.217	0.33	30	118	82
C(3)	0.151	0.204	0.256	0.36	21	110	95
C(4)	0.149	0.205	0.234	0.34	24	114	85
C(5)	0.149	0.210	0.213	0.33	26	92	64
C(6)	0.146	0.191	0.221	0.33	32	120	80
N	0.146	0.184	0.221	0.32	144	61	71

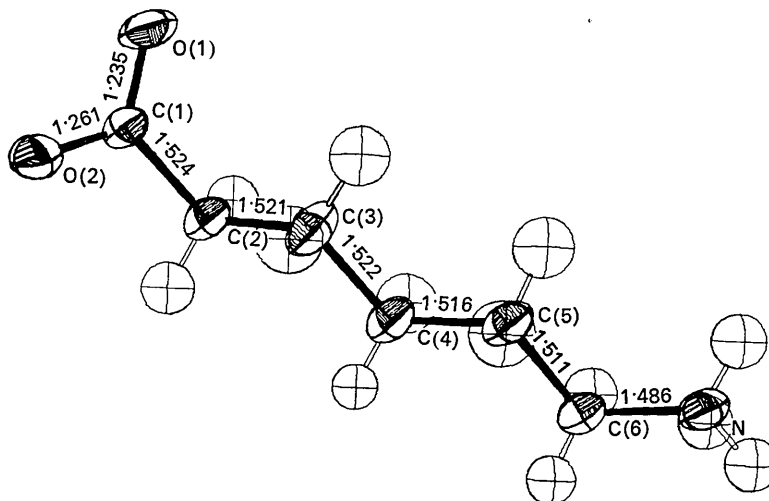
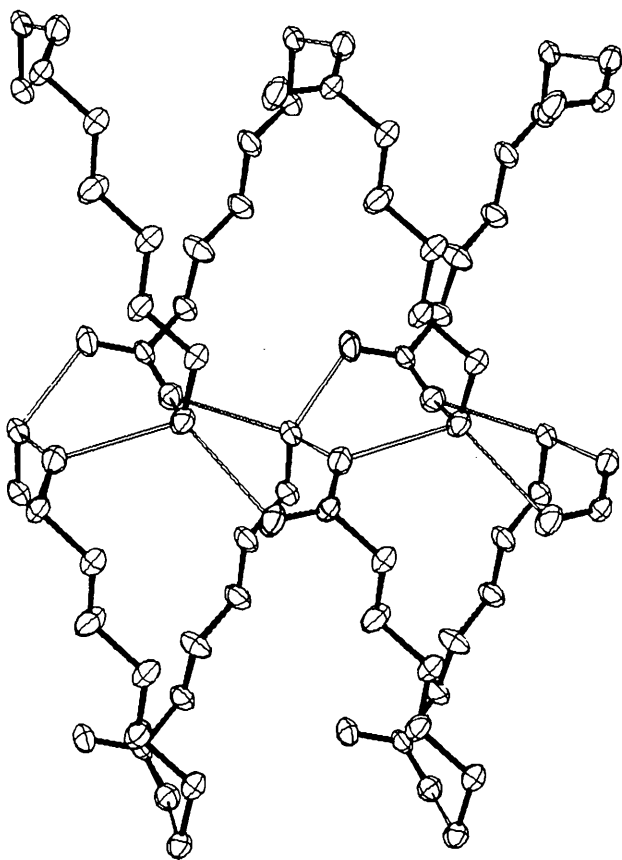
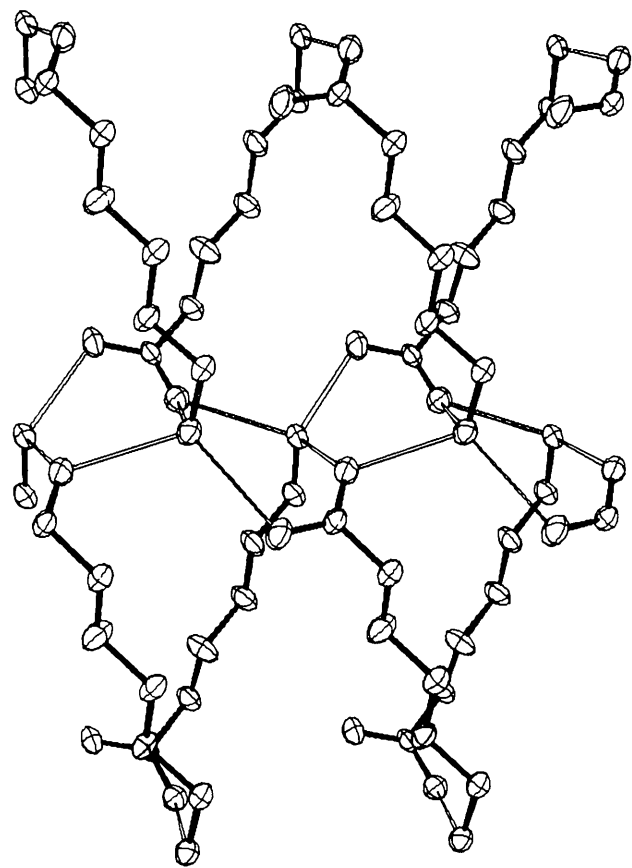


Fig. 1. Schematic drawing of the ϵ -aminocaproic acid molecule.



EPSILON-AMINOCAPROIC ACID

EPSILON-AMINOCAPROIC ACID

Fig. 2. Stereo pair view along the *c* axis.

pared with a normal carbon–nitrogen single bond length of 1.47 Å. In ϵ -aminocaproic acid this distance is 1.485 ± 0.005 Å; in the structures mentioned above, Marsh and co-workers reported corresponding values of 1.480, 1.484 and 1.491 Å. In view of the relatively high accuracy of these later values, it seems likely that the lengthening of this bond above the normal value is a real effect but one which is substantially smaller than indicated by the values quoted by Hahn.

The three hydrogen bonds formed by each nitrogen atom are: 2.753 [N–O(1)], 2.728 [N–O(2)] and 2.814 Å [N–O(2')], all ± 0.005 Å. It is evident that the fact that O(2) participates in two hydrogen bonds, while O(1) is involved in only one, does not affect the lengths of the hydrogen bonds significantly.

The average distance between hydrogen atoms bonded to the same carbon atom is 1.59 ± 0.065 Å; the corresponding average distance among the hydrogen atoms in the NH_3^+ group is 1.55 ± 0.065 Å.

Bond angles in the molecule are 'normal', *i.e.* similar to those reported for molecules of this general type. The average C–C–C angle is 112.2° .

The spatial arrangement of the non-hydrogen atoms is conveniently described in terms of the dihedral angles between the planes of adjacent triatomic groups. Thus, the angle between O(1), O(2), C(1) and C(1), C(2), C(3) is 98.4° ; it is 7.6° between C(1), C(2), C(3) and C(3), C(4), C(5) and 5.6° between the latter group and C(5), C(6), N ... all $\pm 0.2^\circ$.

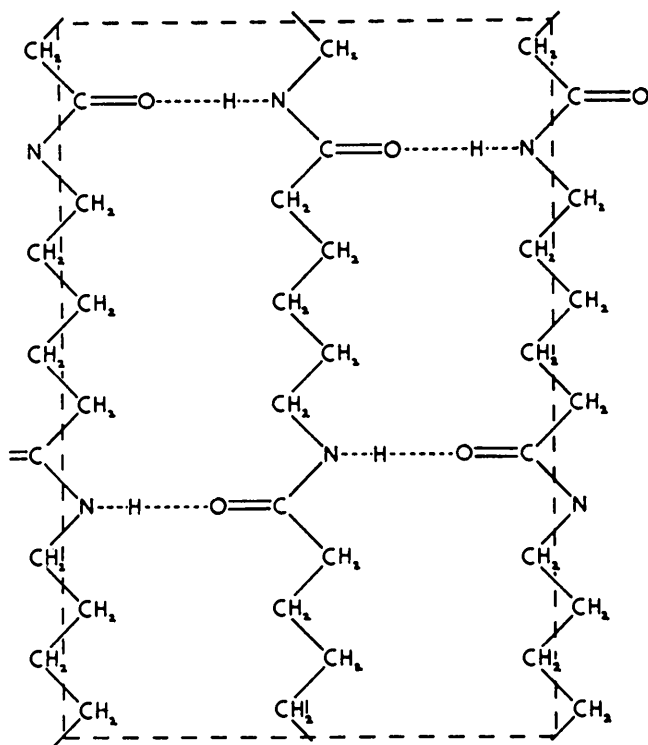


Fig. 3. Molecular layer in Nylon-6.

Molecular packing

A stereo pair drawing (Johnson, 1965) of a portion of the unit cell, viewed along *c*, is shown in Fig. 2. Two sets of molecular chains may be distinguished; one slopes down to the right along [110] and is inclined approximately $27\frac{1}{2}^\circ$ to the horizontal axis *a*; the other is oriented along [1 $\bar{1}$ 0]. Each set lies in one layer. There are four such layers in each unit cell, all roughly perpendicular to *c*.

Each nitrogen atom forms three hydrogen bonds; two link it with oxygen atoms [O(1) and O(2')] in molecules in the layer containing the nitrogen atom. The third bond is directed to an oxygen atom O(2') in an adjacent layer.

Hydrogen bonds of the latter type link the layer at $z = \frac{1}{8}$ to that at $z = \frac{3}{8}$ and the layer at $z = \frac{5}{8}$ to that at $z = \frac{7}{8}$, forming strongly bonded double layers. These in turn are held together by relatively weak intermolecular interactions.

The solid state conversion to Nylon-6

We have mentioned above that ϵ -aminocaproic acid undergoes polycondensation in the solid state. Under the combined influence of heat and high vacuum single crystals of the monomer transform to highly crystalline, oriented Nylon-6. It is possible to relate some structural features of the latter to the crystal structure of the monomer.

The crystal structure of Nylon-6 has been determined by Holmes, Bunn & Smith (1955). The unit cell is monoclinic with $a = 9.56$, $b = 17.2$, $c = 8.01$ and $\beta = 112.5^\circ$; the space group is $P2_1$ and there are eight 'condensed' monomer units in the unit cell. The structure may be described in terms of polymeric chains linked to one another by N–H–O bonds to form layers parallel to *bc*. The chains are parallel to *b* (Fig. 3).

Examination of Fig. 3 reveals that the C=O, N, C sequence in one chain is reversed in successive chains within the layer. It will be recalled that in the monomer the reactive groups (*i.e.* carboxyl and NH_3^+) and the carbon atoms are arranged in the same sequence in all molecules in a given layer. The sequence is reversed in adjacent layers. It is evident that individual polymeric chains of Fig. 3 are derived from molecules lying within one layer in the crystal. At some stage the hydrogen bonds are broken and water is split out, leaving one hydrogen atom bonded to each nitrogen, and one oxygen atom in a carbonyl group. The carbon atom to which the oxygen is bonded [C(1)] forms a single bond link with the nitrogen atom.

Adjacent chains within one layer in Fig. 3 are derived from adjacent layers of the monomeric crystal. We have noted that the latter not only differ from one another in atomic sequence, but are inclined to each other by 55° . In the course of the polycondensation process these chains must change their relative orientations to become parallel to one another and to link up with one another by hydrogen bonding as shown in Fig. 3.

Recent electron microscope studies of crystalline Nylon-6 specimens prepared from single crystals of the monomer reveal that twinning is of frequent occurrence in the polymer crystals (Fischer, 1965). The twinning appears to involve rotation of successive layers, or groups of layers, of the type shown in Fig. 3, by 55° relative to one another. Such twinning is not observed in Nylon specimens prepared from the melts (Macchi, 1966). Although the detailed mechanism of the twinning is not known, it is not difficult to relate the 55° twin angle of the polymer to the 55° by which molecular chains in successive layers of the monomer are inclined to one another.

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The Crystal Structure of Thiodiglycollic Acid*

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Thiodiglycollic acid, $S(CH_2COOH)_2$, crystallizes in space group *Pnam* with 4 molecules in the unit cell of dimensions $a = 5.050 \pm 0.005$, $b = 6.701 \pm 0.003$, $c = 17.742 \pm 0.004$ Å. The structure has been solved from two-dimensional photographic data and has been refined by least squares with three-dimensional diffractometer data. The structure consists of parallel infinite chains of hydrogen bonded molecules. Hydrogen bonding takes place between carboxyl groups around the centre of symmetry; they are coplanar within experimental error.

Introduction

Thiodiglycollic acid, $S(CH_2COOH)_2$ (also known as methyl sulphide α,α -dicarboxylic acid or thiodiacetic acid), is of importance because of its use as a precipitating organic reagent for zirconium (Sant & Sant, 1959). It is used for the detection of copper, lead, mercury and silver, and also for the estimation of cadmium.

A preliminary report of the structure has already been published (Roy, 1965).

Experimental

Single crystals of thiodiglycollic acid were grown by slow evaporation of a concentrated aqueous solution

at room temperature. The crystals appeared as thin plates elongated in the [100] direction. The unit-cell dimensions determined by means of a General Electric Single Crystal Orienter are: $a = 5.050 \pm 0.005$, $b = 6.701 \pm 0.003$, $c = 17.742 \pm 0.004$ Å.

Systematic absences as found on Weissenberg photographs about the *a* and *b* axes were consistent with the space groups *Pnam* and *Pna2₁*. The density as determined by flotation is 1.66 g.cm^{-3} , and the calculated density for four molecules per unit cell is 1.70 g.cm^{-3} .

The structure was solved from the two projections along [100] and [010]. The *Ok_l* and *h0_l* intensities were estimated visually from the integrated Weissenberg photographs. The three-dimensional data used for the refinement of the structure were collected with a General Electric diffractometer equipped with a scintillation counter using β -filtered Cu $K\alpha$ radiation and discrimination.

Peak intensities were measured which were converted to integrated intensities in the following way. For a set of 30 reflexions of θ values, distributed over the accessible θ range, both peak intensities and inte-

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